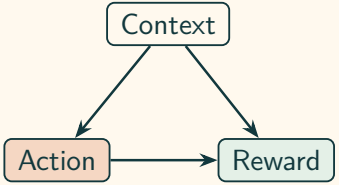
Team Ghost reading group

An Introduction to Causal Reasoning



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Which delivery service would you choose?

You need to choose a service for your next delivery. Both cost the same. You want the delivery to arrive on time.

Service A

63%

6,300 of 10,000 past deliveries on time

Service B

77%

7,700 of 10,000 past deliveries on time

Based on these results, pick one service.

Service A

Service B

Would you change your vote?

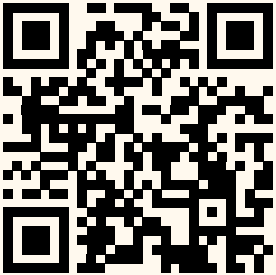
Overall, B led A, **77%** to 63%. Here are the same deliveries split by route length.

Short routes			Long routes		
Service A	**90%**	(900/1,000)	Service A	**60%**	(5,400/9,000)
Service B	80%	(7,200/9,000)	Service B	50%	(500/1,000)

Vote again based on the breakdown. Pick one service.

Service A | **Service B**

Who wants to try the causal tablet?

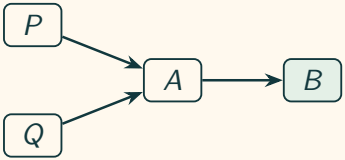


https://cyvernes.github.io/tablette.html

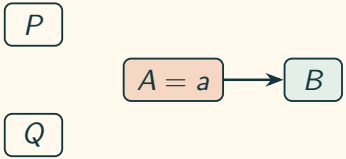
From the tablet to a causal graph

Nodes represent variables. **Arrows** represent direct causal effects.

Original graph



After $\mathrm{do}(A = a)$



The intervention $\mathrm{do}(A = a)$ removes every arrow from a parent of A into A.

A reminder on d-separation

What happens to the path from X to Y when we condition on Z?

Fork



Leave Z out $\rightarrow$ open
Condition on Z $\rightarrow$ blocked

Collider



Leave Z out $\rightarrow$ blocked
Condition on Z $\rightarrow$ open

Chain

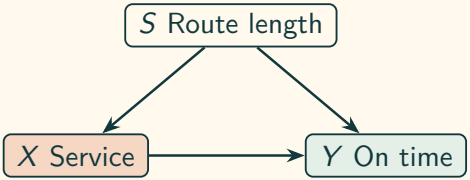


Leave Z out $\rightarrow$ open
Condition on Z $\rightarrow$ blocked

Conditioning on a descendant of a collider also opens the path at that collider.

Let $\mathbf{X}, \mathbf{Y}, \mathbf{W}$ be pairwise disjoint sets of variables in a causal graph $\mathcal{G}$.
We write $\mathbf{X} \perp\!\!\!\perp_{\mathcal{G}} \mathbf{Y} \mid \mathbf{W}$ *iff* **every path** from $\mathbf{X}$ to $\mathbf{Y}$ is blocked given $\mathbf{W}$.

Back to the delivery vote

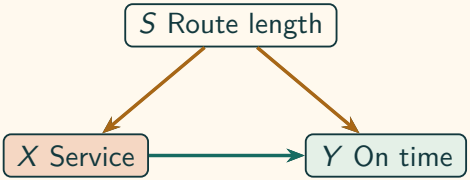


Service B handled most deliveries on short routes. Service A handled most deliveries on long routes.

Route length also affects punctuality.

Which paths connect the logged service X to punctuality Y?

Back to the delivery vote



Service B handled most deliveries on short routes. Service A handled most deliveries on long routes.

Route length also affects punctuality.

Causal path $X \to Y$

The service can affect punctuality.

Noncausal path $X \leftarrow S \to Y$

Seeing which service was used gives a clue about route length, which also predicts punctuality.

Compare deliveries of the same route length

Condition on route length S to block the noncausal path $X \leftarrow S \rightarrow Y$. Compare the on-time rates within each route type.

Short routes		Long routes	
90% for A	80% for B	**60%** for A	50% for B

Choose Service A. It has the higher on-time rate for both short and long routes.

How can we formalize this?

Pearl's do-calculus

Let $\mathcal{G}$ be a causal graph and let $\mathbf{X}, \mathbf{Y}, \mathbf{Z}, \mathbf{W}$ be pairwise disjoint sets of variables. $G_{\overline{\mathbf{X}}}$ deletes arrows into $\mathbf{X}$ and $G_{\underline{\mathbf{Z}}}$ deletes arrows out of $\mathbf{Z}$.

$$\mathcal{R}_1 \quad P(\mathbf{y} \mid \mathrm{do}(\mathbf{x}), \mathbf{z}, \mathbf{w}) = P(\mathbf{y} \mid \mathrm{do}(\mathbf{x}), \mathbf{w}) \qquad \text{if } \mathbf{Y} \perp\!\!\!\perp_{G_{\overline{\mathbf{X}}}} \mathbf{Z} \mid \mathbf{X}, \mathbf{W},$$

$$\mathcal{R}_2 \quad P(\mathbf{y} \mid \mathrm{do}(\mathbf{x}, \mathbf{z}), \mathbf{w}) = P(\mathbf{y} \mid \mathrm{do}(\mathbf{x}), \mathbf{z}, \mathbf{w}) \qquad \text{if } \mathbf{Y} \perp\!\!\!\perp_{G_{\overline{\mathbf{X}}, \underline{\mathbf{Z}}}} \mathbf{Z} \mid \mathbf{X}, \mathbf{W},$$

$$\mathcal{R}_3 \quad P(\mathbf{y} \mid \mathrm{do}(\mathbf{x}, \mathbf{z}), \mathbf{w}) = P(\mathbf{y} \mid \mathrm{do}(\mathbf{x}), \mathbf{w}) \qquad \text{if } \mathbf{Y} \perp\!\!\!\perp_{G_{\overline{\mathbf{X}}, \overline{\mathbf{Z(W)}}}} \mathbf{Z} \mid \mathbf{X}, \mathbf{W}.$$

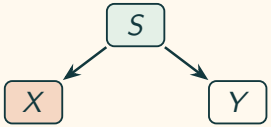
Here $\mathbf{Z(W)} = \mathbf{Z} \setminus \mathrm{An}_{G_{\overline{\mathbf{X}}}}(\mathbf{W})$. The equalities apply where everything is defined.

Pearl, *Causality*, 2nd ed. (2009), §3.4.2, Theorem 3.4.1

Which two rules justify the adjustment?

X is the delivery service, S the route length, and Y the indicator of an on-time delivery.

$\mathcal{R}_2$ **in** $G_{\underline{X}}$

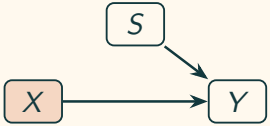


Conditioning on S blocks the fork.

$$Y \perp\!\!\!\perp X \mid S$$

$$P(y \mid \mathrm{do}(x), s) = P(y \mid x, s)$$

$\mathcal{R}_3$ **in** $G_{\bar{X}}$



The collider Y blocks the path.

$$S \perp\!\!\!\perp X$$

$$P(s \mid \mathrm{do}(x)) = P(s)$$

Deriving the back-door formula

To get the population-wide on-time rate, sum over route length S.

$$
\begin{aligned}
P(y \mid \mathrm{do}(x)) &= \sum_{s} P(y, s \mid \mathrm{do}(x)) && \text{(marginalization)} \\
&= \sum_{s} P(y \mid \mathrm{do}(x), s)\, P(s \mid \mathrm{do}(x)) && \text{(product rule)} \\
&= \sum_{s} P(y \mid x, s)\, P(s \mid \mathrm{do}(x)) && (\mathcal{R}_2) \\
&= \sum_{s} P(y \mid x, s)\, P(s). && (\mathcal{R}_3)
\end{aligned}
$$

A back-door set contains no descendants of X and blocks every path from X to Y that starts with an arrow into X. Here S qualifies.

Which service should we choose?

The route length S is fixed for this delivery. Compare the on-time rates under $\text{do}(X = A)$ and $\text{do}(X = B)$ for each possible length.

Route length S	**Choose A**	**Choose B**
Known short route	**90%**	80%
Known long route	**60%**	50%
Unknown	$\frac{1}{2}(90\%) + \frac{1}{2}(60\%) = \mathbf{75\%}$	$\frac{1}{2}(80\%) + \frac{1}{2}(50\%) = 65\%$

Choose A in all cases.

In what order should we try the rules?

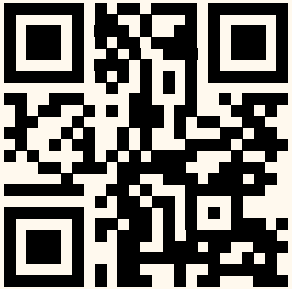
Do-calculus reminder

Each rule rewrites the query when its graphical separation test holds.

$\mathcal{R}_1$	Add or remove an observation	$P(\mathbf{y} \mid \mathrm{do}(\mathbf{x}), \mathbf{z}, \mathbf{w}) = P(\mathbf{y} \mid \mathrm{do}(\mathbf{x}), \mathbf{w})$
$\mathcal{R}_2$	Swap an observation and an intervention	$P(\mathbf{y} \mid \mathrm{do}(\mathbf{x}), \mathbf{z}, \mathbf{w}) = P(\mathbf{y} \mid \mathrm{do}(\mathbf{x}, \mathbf{z}), \mathbf{w})$
$\mathcal{R}_3$	Add or remove an intervention	$P(\mathbf{y} \mid \mathrm{do}(\mathbf{x}, \mathbf{z}), \mathbf{w}) = P(\mathbf{y} \mid \mathrm{do}(\mathbf{x}), \mathbf{w})$

Each rule changes the query and can change the graphical tests for the next step. This makes it hard to know in advance which sequence will work.

Let’s explore derivation graphs



https://lig-causaforge.imag.fr/

Which rule applications commute?

A Rule 1 move can be replaced by a Rule 2 move and a Rule 3 move.

Up $\uparrow$ adds an intervention or turns an observation into one.
Down $\downarrow$ drops an intervention or turns it into an observation.

Consecutive moves on disjoint sets of variables

Moves	**Same direction**	**Opposite directions**
$\mathcal{R}_2,\ \mathcal{R}_2$	Swap freely	Swap freely
$\mathcal{R}_3,\ \mathcal{R}_3$	Swap freely	$\mathcal{R}_3^{\downarrow}\mathcal{R}_3^{\uparrow} \Rightarrow \mathcal{R}_3^{\uparrow}\mathcal{R}_3^{\downarrow}$
$\mathcal{R}_i,\ \mathcal{R}_j,\ \{i,j\} = \{2,3\}$	Swap freely	$\mathcal{R}_i^{\downarrow}\mathcal{R}_j^{\uparrow} \Rightarrow \mathcal{R}_j^{\uparrow}\mathcal{R}_i^{\downarrow}$

Yvernes et al., Theorems 2–5

Every valid path has a normal form

The two queries determine the four blocks below from their observed and intervened variables. If any valid derivation connects the queries, this sequence connects them too.

$\mathcal{R}_2^{\uparrow}$	$\mathcal{R}_3^{\uparrow}$	$\mathcal{R}_3^{\downarrow}$	$\mathcal{R}_2^{\downarrow}$
observe $\rightarrow$ act	add action	drop action	act $\rightarrow$ observe

Each block can act on several variables. Skip empty blocks.

Yvernes et al., Theorem 6

Are the endpoint queries equivalent?

For two queries with the same outcome in a fixed causal graph, test each nonempty block for d-separation in its corresponding modified graph.

Corollary 1. The queries are equivalent under do-calculus exactly when all tests pass. At most four tests are needed. If one fails, no sequence of do-calculus rules connects the queries.

Delivery example

$$P(y \mid \mathrm{do}(x), s) \xrightarrow{\mathcal{R}_2^{\downarrow}(X)} P(y \mid x, s)$$

Only X changes status, so there is one test. After removing arrows out of X, conditioning on S blocks the fork $X \leftarrow S \rightarrow Y$.

Yvernes et al., Corollary 1

Can we identify the effect?

Given the graph, **identifiable** means computable from observations alone.

- **ID** finds a do-free formula for $P(\mathbf{y} \mid \mathrm{do}(\mathbf{x}))$ if and only if the effect is identifiable.
- **IDC** gives the same guarantee for conditional queries $P(\mathbf{y} \mid \mathrm{do}(\mathbf{x}), \mathbf{w})$.

Live demo **https://lig-causaforge.imag.fr/**

Shpitser and Pearl (2008), identification algorithms

Conclusion and discussion

- Causal graphs make our assumptions explicit, and help us reason causally.
- Do-calculus uses graphical separation tests to rewrite causal queries.
- For two queries with the same outcome, their endpoints determine a sequence of at most four tests for equivalence under do-calculus.
- ID and IDC decide identifiability and return a do-free formula when one exists.

Thank you for taking part! Questions and discussion

References and live tools

Core references

Pearl (2009), *Causality*, 2nd ed., Ch. 3 (back-door adjustment and do-calculus).

Yvernes et al. (2026), *Do-Calculus Derivation Graphs*, §§3–4, Thms. 2–6, Cor. 1.

Yvernes et al. (2026), *Identifiability in Causal Abstractions*, §2.2, Def. 1.

Shpitser & Pearl (2008), *Complete Identification Methods*, Figs. 4 and 7 (ID and IDC).

Further reading on front-door policy evaluation

Bareinboim, Zhang & Lee, *Causal Reinforcement Learning*, §4.3.2, Examples 37 and 40.

Live tools

Causal tablet CausaForge