Identifiability in Causal Abstractions: A Hierarchy of Criteria

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Causal Graph Abstractions

- Full causal diagrams are rarely known in practice.
- Causal abstractions provide simplified models encoding partial causal structure.
- Each abstraction defines a collection $\mathcal{C}$ of compatible causal graphs over the variables $\mathcal{V}$.
- A query $P(\mathbf{y} \mid \mathrm{do}(\mathbf{x}), \mathbf{w})$ is identifiable in $\mathcal{C}$ if all $\mathcal{G} \in \mathcal{C}$ yield the same estimand.

Too Long; Didn't Read

- We study identifiability over sets of graphs from causal abstractions.
- Multiple definitions exist—we compare and formalize them.
- A conjecture remains open.

Common Backdoor

There exists $\mathcal{Z} \subseteq \mathcal{V}$ that satisfies the back-door criterion in every $\mathcal{G} \in \mathcal{C}$. [1]

. . .

Common Frontdoor

There exists $\mathcal{Z} \subseteq \mathcal{V}$ that satisfies the front-door criterion in every $\mathcal{G} \in \mathcal{C}$. [7]

Common Graphical Criterion

There exists a graphical criterion that holds in every $\mathcal{G} \in \mathcal{C}$.

Backdoor criterion is not complete in a single graph

Frontdoor criterion is not complete in a single graph

Common Do-Calculus (ICD)

There exists a do-calculus proof that is valid in every $\mathcal{G} \in \mathcal{C}$. [3]

- Atomically complete calculus [2,4]

ICD vs IG Conjecture

There exists a class $\mathcal{C}$ and a query Q such that Q is IG in $\mathcal{C}$ but Q is *not* ICD in $\mathcal{C}$.

- No example is known yet

If True: Deriving an atomically complete calculus in a causal abstraction is not enough for solving identifiability.
If False: Any atomically complete calculus is complete

?

Identifiability through Graphs (IG)

There exists a causal estimand valid for any SCM inducing $\mathcal{G}_\mathcal{M} \in \mathcal{C}$.

- Usual Notion of identifiability [2,4,8]

Consider $\mathcal{C} = \{X \to Y,\ X \leftarrow Y\}$. $P(y \mid \mathrm{do}(x))$ is not IG in $\mathcal{C}$. However, if $X \perp\!\!\!\perp_{P^\star} Y$, then $P(y \mid \mathrm{do}(x))$ is IGP in $\mathcal{C}$.

Identifiability through Graphs Knowing $P^\star$ (IGP)

There exists a causal estimand valid for any SCM $\mathcal{M}$ inducing $\mathcal{G}_\mathcal{M} \in \mathcal{C}$ and $P_\mathcal{M} = P^\star$.

- An example of structural hypothesis [9]

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