Complete Characterization for Adjustment in Summary Causal Graphs of Time Series

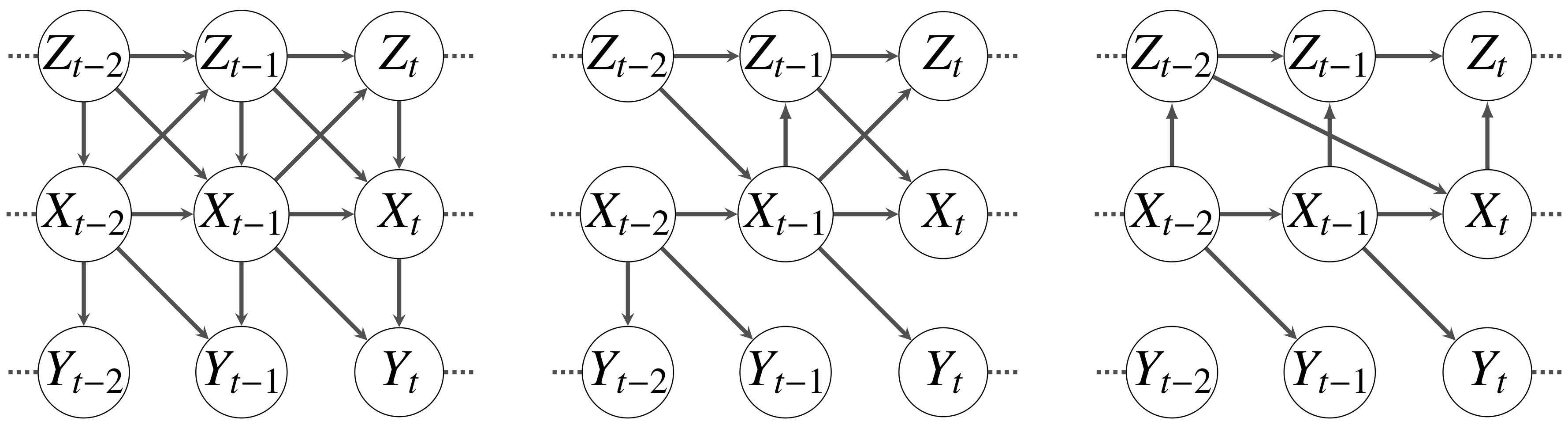
Clément Yvernes [1] Emilie Devijver [1] Eric Gaussier [1]

[1]Univ Grenoble Alpes, CNRS, Grenoble INP, LIG

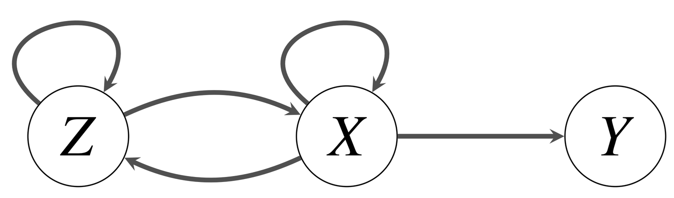
Too Long; Didn't Read

Identifiability by common adjustment in summary causal graphs can be decided in pseudo-linear time complexity.

Causal Graphs in Time Series



Three Full Time Causal Graphs (FTCGs), $\mathcal{G}^f_1$, $\mathcal{G}^f_2$ and $\mathcal{G}^f_3$.



The Summary Causal graph (SCG) $\mathcal{G}^s$, reduced from any FTCG in (a).

An SCG corresponds to a family of candidate FTCGs, denoted $\mathcal{C}(\mathcal{G}^s)$.

Assumptions

- Causal Sufficiency
- **Consistency Through Time:** Results are given both with and without this assumption.

Identifiability by Common Adjustment

In a given SCG $\mathcal{G}^s$, the total effect $P\left(y_t \mid \operatorname{do}\left(x^i_{t-\gamma_i}\right)_i\right)$ is identifiable by common adjustment in $\mathcal{G}^s$ if there exists a subset $\mathbf{Z}$ of $\mathcal{V}^f$ such that for any density P compatible with any candidate FTCG $\mathcal{G}^f \in \mathcal{C}(\mathcal{G}^s)$, we have:

$$P\left(y_t \mid \operatorname{do}\left(x^i_{t-\gamma_i}\right)_i\right) = \begin{cases} P\left(y_t \mid (x^i_{t-\gamma_i})_i\right), & \text{if } \mathbf{Z} = \emptyset, \\ \sum_{\mathbf{z}} P(y_t \mid (x^i_{t-\gamma_i})_i, \mathbf{z})\, P(\mathbf{z}), & \text{otherwise.} \end{cases}$$

Property: A set $\mathbf{Z} \subseteq \mathcal{V}^f$ satisfies the common adjustment criterion relative to $\mathcal{X}^f := \left\{X^i_{t-\gamma_i}\right\}_i$ and Y_t in $\mathcal{G}^s$ if, for every full-time causal graph $\mathcal{G}^f \in \mathcal{C}(\mathcal{G}^s)$, we have:

1. $\operatorname{Forb}\left(\mathcal{X}^f, Y_t, \mathcal{G}^f\right) \cap \mathbf{Z} = \emptyset$; and
2. $\mathbf{Z}$ blocks all proper non-causal paths from $\mathbf{X}$ to $\mathbf{Y}$ in $\mathcal{G}$.

where the forbidden set $\operatorname{Forb}\left(\mathcal{X}^f, Y_t, \mathcal{G}^f\right)$ is the set of all descendants of any $W \notin \mathcal{X}^f$ which lies on a proper causal path from $\mathcal{X}^f$ to Y_t.

Adjustment in SCGs

Common Forbidden Set: $\mathcal{CF} := \bigcup_{\mathcal{G}^f \in \mathcal{C}(\mathcal{G}^s)} \operatorname{Forb}\left(\mathcal{X}^f, Y_t, \mathcal{G}^f\right)$

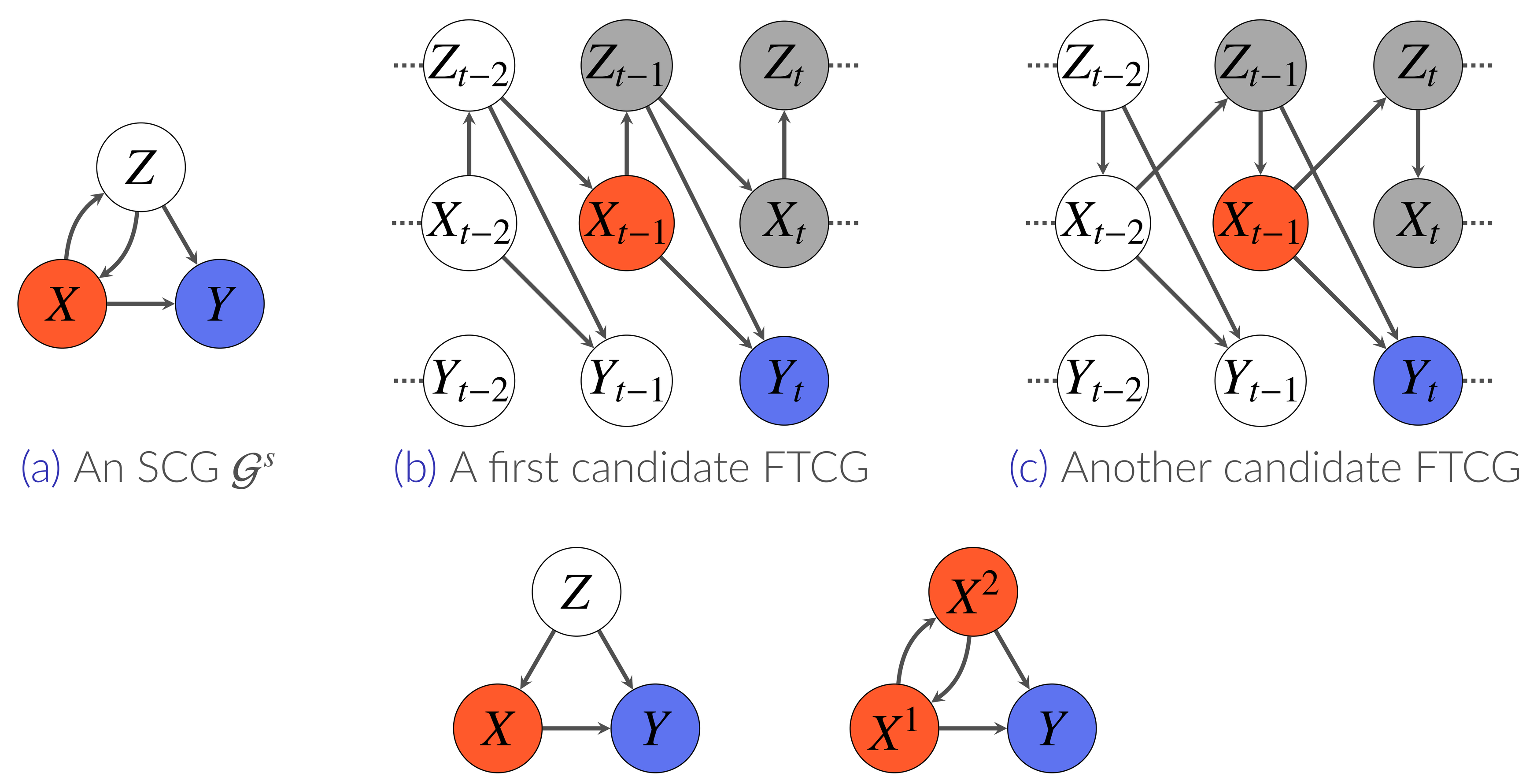
Non-Conditionable Variables: $\mathcal{NC} := \mathcal{CF} \setminus \mathcal{X}^f$

Remark. The set $\mathcal{NC}$ characterizes variables that must be excluded from adjustment sets when estimating the causal effect.

Main Theorem. The total effect is identifiable by common adjustment in $\mathcal{G}^s$ if and only if the following condition holds:

- For all $X^i_{t-\gamma_i}$ and all $\mathcal{G}^f \in \mathcal{C}(\mathcal{G}^s)$, $\mathcal{G}^f$ does not contain any collider-free backdoor path going from $X^i_{t-\gamma_i}$ to Y_t that remains in $\mathcal{NC} \cup \{X^i_{t-\gamma_i}\}$.

In that case, a common adjustment set is given by $\mathcal{C} := \left(\mathcal{V}^f \setminus \mathcal{NC}\right) \setminus \mathcal{X}^f$.



(a) An SCG $\mathcal{G}^s$ (b) A first candidate FTCG (c) Another candidate FTCG

(d) An SCG $\mathcal{G}^s$ (e) An SCG $\mathcal{G}^s$

Figure 2. Consider $P(y_t \mid \operatorname{do}(x_{t-1}))$ in (a), where the total effect is not identifiable by common adjustment. In contrast, $P(y_t \mid \operatorname{do}(x_{t-1}))$ is identifiable in (d), and $P(y_t \mid \operatorname{do}(x^1_{t-1}, x^2_{t-1}))$ is identifiable in (e). Orange vertex: the variable we intervene on, blue vertex: the response we are considering, gray vertices: elements of $\mathcal{NC}$.

Algorithmic Characterization

- The condition in the Main Theorem can be decided by a sound and complete algorithm.
- The algorithm runs in pseudo-linear complexity.

Discussion & Perspectives

- Remove causal sufficiency
- Towards a sound and complete calculus in SCGs
- Algorithmic approaches may be key to tackling more complex forms of causal abstraction

References

- Anand, T. V., Ribeiro, A. H., Tian, J., and Bareinboim, E.(2023). Causal effect identification in cluster DAGS. AAAI Conference.
- Assaad, C. K., Devijver, E., Gaussier, E., Goessler, G., and Meynaoui, A. (2024). Identifiability of total effects from abstractions of time series causal graphs. UAI 2024.
- Perkovic, E., Textor, J., Kalisch, M., and Maathuis, M. H. (2016). Complete graphical characterization and construction of adjustment sets in markov equivalence classes of ancestral graphs. JMLR.

clement.yvernes@univ-grenoble-alpes.fr